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replacing the values of the functions considered at a and b with their values at a+0 and b-0. In this way the result of Marchand can be improved. Further improvements are pointed out for Sylvester's theorems and Newton's Rule.

The reversible transformation of space elements can be characterized, starting from ordinary one-to-one transformations between two convenient spaces and the complete sets of integrals of certain partial differential equations, which can be expressed by convenient determinants.

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I. The paper contains a simplified proof of a sharpening of Sylvester's theorems, obtained by replacing the values of the functions considered at a and b with their values at a+0 and b-0. In this way the result of Marchand can be improved. Further improvements are pointed out for Sylvester's theorems and Newton's Rule.

II. The reversible transformation of space elements can be characterized, starting from ordinary one to one transformations between two convenient spaces and the complete sets of integrals of certain partial differential equations, which can be expressed by convenient determinants.

I. On Newton-Sylvester's theorems

1. For

$$f_o(x) = a_o x^n + a_1 x^{n-1} + \dots + a_n \ , \ a_o = 1 \tag{1}$$

form the expressions

$$\begin{aligned} F_\nu(x) &:= r_\nu f^{(\nu)2}(x) - r_{\nu-1} f^{(\nu-1)}(x) f^{(\nu+1)}(x) \quad (\nu=1,\dots,n-1) \ , \\ F_o(x) &:= f_o(x)^2 \ , \quad F_n(x) := 1 \ , \end{aligned} \tag{2}$$

assuming that the positive constants r_ν satisfy the relations

$$r_{\nu+1} = 2r_\nu - r_{\nu-1} \quad (\nu=1,\dots,n-2) \ . \tag{3}$$

Denote by $VP(x_o)$ the number of indices ν with

$$\operatorname{sgn}\left(f^{(\nu-1)}(x_o) f^{(\nu)}(x_o)\right) = -1 \ , \ \operatorname{sgn}\left(F_{\nu-1}(x_o) F_\nu(x_o)\right) = +1 \ .$$

These are <u>variation permanences</u> corresponding to ν and x_o. Similarly denote by $PP(x_o)$ the number of indices ν with

$$\operatorname{sgn}\left(f^{(\nu-1)}(x_o) f^{(\nu)}(x_o)\right) = \operatorname{sgn}\left(F_{\nu-1}(x_o) F_\nu(x_o)\right) = +1 \ .$$

Here we have <u>permanence permanences</u> corresponding to ν and x_o.

Denoting by $N_m(a,b)$ the number of zeros of $f^{(m)}(x)$ in (a,b), Sylvester's theorems say that the expressions

$$VP(a) - VP(b) - N_o(a,b) \ , \tag{4}$$

$$PP(b) - PP(a) - N_o(a,b) \ , \tag{5}$$

are <u>non-negative and even</u>.

2. The proofs given by Sylvester, Genocchi and de Jonquières were rather incomplete and only Marchand (1913) gave a complete discussion of the theorems in their original form. Difficulties arrose mainly about the right interpretation of zeros in the double sequence of $f^{(\nu)}, F_\nu$ for a and b. In this respect the solution given by Marchand is not optimal since the optimal results can be only obtained for the open interval (a,b) considering the numbers of VP and PP at a+0 and b-0. Thus, asking for convenient attribution of the signs at a, b themselves is practically asking the wrong question.

3. In this paper a generalization of Sylvester's theorems is derived assuming only that $f_o^{(\nu)}(x)$ $(\nu=0,\dots,n)$ are continuous functions of x in $\langle a,b\rangle$ with finite numbers of zeros in $\langle a,b\rangle$, while $f^{(n)}$ does not vanish in (a,b). We further have to assume that each of the F_ν formed accordingly to (2) for a fixed choice of the r_ν either identically vanishes or has there only a finite number of zeros. Identically vanishing F_ν are assumed to be provided with the plus sign.

Defining for $a<b$ the interval functions

(6) $$\Delta'_m(a,b) \quad := \quad VP_m(a+0) - VP_m(b-0) \quad ,$$

(7) $$\Delta''_m(a,b) \quad := \quad PP_m(b-0) - PP_m(a+0) \quad ,$$

we can formulate our main result as saying that the differences

(8) $$\Delta'_m(a,b) - \Big(N_o(a,b)-N_m(a,b)\Big) \quad , \quad \Delta''_m(a,b) - \Big(N_o(a,b)-N_m(a,b)\Big)$$

are non-negative even numbers.

We further give the explicit solution of the problem of obtaining the signs at a+0 and at b-0 for the values of functions

at a, b and derive as corollaries of our main result corresponding generalizations of Newton's Rule. The paper contains as compared with Marchand's presentation an essentially simplified presentation of the whole subject.

II. Reversible transformations of space elements

Consider, for $n > 1$, the coordinates, $x_1, \ldots, x_n$, of a general point of the n-dimensional space, depending on and arbitrarily often differentiable with respect to m parameters $T_1, \ldots, T_m$. Denote generally the derivatives $\frac{\partial x_\nu}{\partial T_\mu}$ by $p_{\nu\mu}$ ($\nu=1,\ldots,n;\mu=1,\ldots,m$).

In this paper we are going to consider the transformation

$$(1) \qquad y_\nu = Y_\nu^*(x_\nu, p_{\nu\mu}) \quad (\nu=1,\ldots,n) \qquad ,$$

where the Y_ν^* are homogeneous of dimension 0 in the $p_{\nu\mu}$ and have the further property:

Differentiating y_ν in (1) with respect to the T_μ and putting

$$q_{\nu\mu} := \frac{\partial y_\nu}{\partial T_\mu}$$

we can, eliminating the $p_{\nu\mu}$ and their derivatives, express the x_ν in function of y_ν and $q_{\nu\mu}$,

$$(2) \qquad x_\nu = X_\nu^*(y_\nu, q_{\nu\mu}) \quad (\nu=1,\ldots,n) \qquad ,$$

where the X^*_ν are homogeneous of dimension 0 in the $q_{\nu\mu}$; and inversely (1) can be deduced differentiating (2) and eliminating the $q_{\nu\mu}$. The functions X^*_ν, Y^*_ν are assumed arbitrarily often differentiable in their arguments.

Such transformations will be called <u>reversible transformations</u>.

We prove that there exist two sets of k functions

$$(3) \qquad r_\varkappa = r^*_\varkappa(x_\nu, p_{\nu\mu}) \quad , \quad s_\varkappa = s^*_\varkappa(y_\nu, q_{\nu\mu}) \quad (\varkappa=1,\dots,k) \;,$$

where each set is independent, and which have the property that the expressions Y^*_ν in (1) and X^*_ν in (2) can be written as

$$(4) \qquad Y^*_\nu =: Y_\nu(x_\nu, r_\varkappa) \quad , \quad X^*_\nu =: X_\nu(y_\nu, s_\varkappa) \quad (\nu=1,\dots,n) \;.$$

Hence, there exists a one to one transformation between two (n+k)-dimensional spaces $(x_\nu, r_\varkappa)$ and $(y_\nu, s_\varkappa)$,

$$(5) \qquad T \begin{cases} y_\nu = Y_\nu(x_\nu, r_\varkappa) \quad , \quad s_\varkappa = S_\varkappa(x_\nu, r_\varkappa) \\ x_\nu = X_\nu(y_\nu, s_\varkappa) \quad , \quad r_\varkappa = R_\varkappa(y_\nu, s_\varkappa) \end{cases} (\nu=1,\dots,n;\varkappa=1,\dots,k) \;.$$

The main problem of the paper is to find characteristic properties of the transformations (5).

The method of the paper consists in studying the system of partial differential equations

$$(6) \qquad J_{\mu,\varkappa}\, U := \sum_{\nu=1}^{n} X'_{\nu s_\varkappa} U'_{p_{\nu\mu}} = 0 \quad (\mu=1,\dots,m;\varkappa=1,\dots,k) \;,$$

$$(7) \qquad \Delta_{\mu,\lambda}\, U := \sum_{=1}^{n} p_{\nu\lambda}\, U'_{p_{\nu\mu}} = 0 \qquad (\mu,\lambda=1,\dots,m) \;,$$

where U is a function of the x_ν, $r_\varkappa$, $p_{\nu\mu}$.

The integrals of these equations are the so called functions with the <u>property U</u>. The maximal number of such independent integrals is

$$N = m(n-k-m+d) \quad .$$

Further, we prove that both systems (6) and (7) are complete as well as the system consisting of (6) and (7) taken together. Further, the system (6) and the system (7) are independent, while both systems taken together are not necessarily so. The discussion depends essentially on the number of independent relations existing between (6) and (7). This number can be denoted by dm, where d has the values 0,1,...,m. But the cases $d>0$ are exceptional.

For d=0 we can write, using k indefinitely often differentiable arbitrary functions,

$$(8) \qquad r_\varkappa = \varphi_\varkappa(U^{(k+1)},\ldots,U^{(k+N)}) \quad (\varkappa=1,\ldots,k) \quad ,$$

where $U^{(\sigma)}$ are N independent integrals of (6) and (7). The $U^{(\sigma)}$ can be formed, using convenient determinants of the $X'_{\nu s_\varkappa}$ and $p_{\nu\mu}$, where the $X'_{\nu s_\varkappa}$ are expressed in x_ν, $r_\varkappa$ and $p_{\nu\mu}$. However, in order to obtain the $r_\varkappa$ solving the k equations (8), certain determinants must be assumed to be $\neq 0$.

As to the exceptional cases $(d>0)$, we give a complete solution for d=1 and d=m, obtained by some special methods. We obtain further, using such methods, the relation

$$(9) \qquad k \leqslant \frac{n-m}{d+1} + \theta \quad , \quad 0 < \theta \leqslant \frac{d}{d+1} \quad .$$

The author was no longer able to discuss the applications of the theory to the discussion of the partial differential equations solvable without integration.